**Simple model**

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| **Parameter** | **Description** | **Quantity** |
|  | Maximal growth rate of strain of species | total # of strains |
|  | Half saturation constant of strain of species | total # of strains |
|  | Death rate of strain of species | total # of strains |
|  | Yield coefficient of strain of species | total # of strains |
|  | Conversion factor of substrate to product for growing D.v. | # D.v. strains |
|  | Product concentration for which there is no growth of strain in D.v. | # D.v. strains |

General Model Description:

We constructed a set of ordinary differential equations to track the change of four dynamic variables: the density of *Desulfovibrio vulgaris* (D)*,* the density of *Methanococcus maripaludis* (M)*,* the concentration of substrate (i.e., lactate) used as an input for *Desulfovibrio vulgaris* metabolism (S), and the product (i.e., hydrogen gas and/or other electron sources) of *Desulfovibrio vulgaris* metabolism that is used as an input for *Methanococcus maripaludis* to perform methanogenesis (P).

Biological Interpretation of Parameters and Assumptions of the Model:

* : (mu) In all cases, μ represents the maximal growth rate of strain *i* of species X. For instance, μDv,SR- (while not an ideal choice since it is fairly clunky) could be used to represent the maximum growth rate of a *Desulfovibrio vulgaris* strain that has lost the capacity to respire sulfate.
* The above parameter is referred to as “maximal” because the actual growth rate of *Desulfovibrio vulgaris* that is growing in coculture will likely vary based on the environment. As the concentration of lactate decreases in the culture, we would expect the growth rate of the organism to also decrease. To reflect this expectation, we use the Monod equation to calculate a realistic growth rate, which is determined by the concentration of substrate. This actual growth rate is expressed in our model by γ (gamma, not to be mistaken with y, which is also used in the model), and is calculated by multiplying the maximal growth rate by a term that quantifies the amount of resource present (R for *Desulfovibrio* is our substrate, S, while R for *Methanococcus* is P).
  + Here, we can see that the maximal growth rate of some hypothetical species is being multiplied by the concentration of resource divided by the same concentration of resource plus a species- and strain- specific constant, k. This constant is known as the half-saturation constant of strain i of species X. It is the concentration of R where the growth rate of the organism is half of the maximal growth rate. To perform a check, plug in k=R into the above equation and you should get R/(R+R) or ½. Another check is that when the concentration of R is 0, we would expect no growth. K is still some positive value (reflecting that you need some positive amount of substrate to reach ½ of your maximal growth rate) but R is now 0. Plugging in 0 for R gives a γ of 0, which we would expect.
  + I found some estimates of Ks for *Desulfovibrio* growing on lactate. Zellner et al. 1994 reported a value of 1.5 mM lactate as the concentration of substrate where *Desulfovibrio* growth is half of maximal while Noguera et al. 1998 estimated this value as 29 mM lactate. I have not found any values reported for Ks of hydrogen for *Methanococcus,* but we can estimate this from experimental data.
* δX,i: (delta) is the death rate of strain i of species X. This will be a relatively simple one to explain for now since we are assuming that the two species do not have an intrinsic rate of death while growing in coculture. This assumption is almost certainly false, but likely not to be all that consequential to our model. By using the Monod equation, we are implementing the characteristic curve that represents the lag, log/exponential, and stationary phases of microbial growth. One limitation is that the model will not predict the death/decline phase of the coculture, but because in the evolution experiment you are frequently transferring these organisms to fresh media the dynamics in this death phase are likely to not matter all that much.
* YX,i: (this is the English letter Y, not to be confused with the Greek letter gamma) is the species- and strain-specific yield coefficient. It serves to convert between the consumption of a resource and the number of new cells that can be produced.
  + Y= mass of new cells/mass of substrate consumed
  + If Y=1 it means that for every 1 unit of substrate, 1 cell is produced. A Y of 0.5 means that it requires 2 units of substrate to produce a single cell.
  + A previous estimate from Traore et al. (1983) stated that the Y of *Desulfovibrio* growing on lactate without an electron acceptor was 5.3 g cell dry mass per mole of lactate.
  + Since this yield coefficient could vary between different variants (if there were a rate/yield trade-off in some of the mutants we explored), we could also estimate this value from experimental data. This estimate would include the assumption that all of lactate in the media were used and we would have to determine the change in the number/mass of cells.
* αD,i: (alpha) is the conversion factor of substrate to product for growing *Desulfovibrio.* An α=1 would mean that for every 1 unit of lactate consumed by *Desulfovibrio,* there is 1 unit of product made available to the methanogen.
  + This is one of the more complex parameters, but also one of the most interesting. It is influenced by unvarying factors (i.e., the stoichiometry of the reaction, where roughly 2 mole hydrogen gas are produced per 1 mole of lactate; Noguera et al. 1998) but also dependent on factors that are more complicated to estimate (e.g., diffusion of hydrogen into the headspace, which could lower the amount of product actually available to be used by the methanogen, or destruction of hydrogen gas by hydrogenases in *Desulfovibrio*).
  + Furthermore, α values could be different between *Desulfovibrio* strains and could indicate how good of a syntrophic partner each variant is. If changes in *Desulfovibrio* (e.g., loss of sulfate respiration) alter the value of this parameter, then the efficiency of the syntrophy changes. For each given unit of lactate consumed by D, there is a different amount of product available to M, which could change the number of M and maybe feedback to alter cell density of both players.
  + Right now, this is going to be a difficult parameter to estimate. My suspicion is that we will start with a value lesser than 1, since from most of my reading I have found that *Desulfovibrio* outnumbers *Methanococcus* throughout most phases of growth. This would be consistent with the idea that for 1 unit of lactate, the methanogen receives less than 1 unit of hydrogen. This goes against the stoichiometry of the reaction, but makes sense given the antagonism that likely exists between the two organisms (with both players capable of consuming hydrogen gas) and the volatile nature of hydrogen (which means that *Methanococcus* is only able to use a subset of the hydrogen gas since much of it will escape to the headspace).
* Pi\*: This is the product concentration for which there is no growth of strain i in *Desulfovibrio.* This value enables a form of product inhibition in the model. The growth of *Desulfovibrio* is modulated by the concentration of resource (according the Monod equation) and is also influenced by 1 minus the current product concentration divided by the constant, P\*, (1-P/P\*). This allows us to slow the growth of *Desulfovibrio* as the concentration of product increases.
  + When the current concentration of product is equal to this threshold product concentration (P=P\*) then 1-(P/P\*) is equal to 0 and there is no growth of *Desulfovibrio.* Continuous fermentation performed by *Desulfovibrio* increases the concentration of products (thereby lowering the thermodynamic favorability of the reaction) and decreases the pH in the coculture, both of which could act to lower the growth rate of *Desulfovibrio.*
  + The inclusion of this inhibition means that if we simulate a population without the methanogen, *Desulfovibrio* would be able to grow up until a threshold concentration of product, at which the culture would plateau in cell density.
  + There are estimates of this value in Noguera et al (1998), but we could also estimate this parameter ourselves if we know the cell density that *Desulfovibrio* reaches when grown without an electron acceptor and without the methanogen.